

Introduction to D-Branes, with Applications

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A brief review of some of the central ideas, terminology and techniques of the technology of orientifolds and D-branes is presented. Some applications are reviewed, including the construction of dual solitonic strings in the context of string/string duality, the computation of the Bekenstein–Hawking entropy/area law for extremal black holes, and the construction of $\mathcal{N}=1$ string vacua in dimensions lower than ten. (Presented at SUSY '96, University of Maryland at College Park, May 29–June 1, 1996.)

1. Overview

Studies of String Theory have traditionally been performed perturbatively[1]. These studies have led to great insight over the years into a rich theory of gravity and possibly the other interactions of the physical world. However, many vital issues have had to be left to await the advent of a non-perturbative understanding of string theory.

Such an understanding has begun to emerge over the last year or so. We are still quite a way (it seems) from addressing real world problems with these new studies, but the time will surely come when we will begin to see if we can understand more about Nature with this approach.

Many of the exciting results from strong coupling string theory have been obtained (or confirmed) with the package of techniques[2] involving ‘**Orientifolds**’[3,4] and ‘**D-Branes**’[5,6].

Generally, D-branes are simply a more precise language with which we can construct perturbative string backgrounds. The advantages come mainly from the fact that certain properties of D-branes allow us to make powerful statements about such perturbative backgrounds which remain true beyond perturbation theory.

2. The Perturbative String Theories

There are five basic superstring theories which may be perturbatively formulated in ten dimensions[1]:

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2.1. The $\mathcal{N}=2$ Theories

The Type IIA and Type IIB theories describe two closed string theories which arise from the choice of how one puts together the right and left moving world-sheet sectors to yield the generators of $\mathcal{N}=2$ spacetime supersymmetry. Choice *A* corresponds to realising a left-moving spinor and a right-moving spinor. Choice *B* corresponds to realising spinors of the same chirality, giving rise to a chiral theory.

2.2. The $\mathcal{N}=1$ Theories

The Heterotic string theories describe closed strings with one supersymmetry from either the left or right moving sector. The other sector is not supersymmetric, and instead has a current algebra which gives rise to spacetime gauge symmetry $SO(32)$ or $E_8 \times E_8$.

The Type I theory is a theory of open strings, with gauge symmetry $SO(32)$. We shall review it in some detail in the next section, and recall how to realise it as the simplest example of an orientifold, involving D-branes.

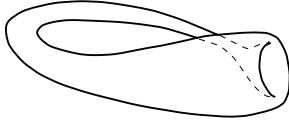
3. Type I from Type IIB: An Orientifold

Recall that for the type IIB theory, there was a symmetry between left and right moving structures on the world sheet. Let us call the generator of this symmetry Ω . It is simply world sheet parity, exchanging left with right.

We can try to construct a new string theory by gauging this symmetry: Project out all states in the theory which are not invariant under Ω . This results in a new closed string theory which is unorientable (the left and right are now

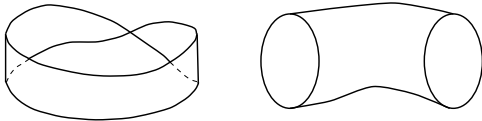
indistinguishable) and so the world sheets for string perturbation theory are unorientable. It is also an $\mathcal{N}=1$ supersymmetric theory, the supersymmetry generator being a linear combination of the two which we started with.

Unfortunately, it is inconsistent. For example, at one loop there is a divergence in the Klein bottle diagram:

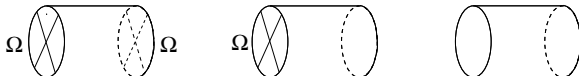


Meanwhile, one might try to construct a purely open $\mathcal{N}=1$ superstring theory, endowing the string endpoints with charges (**‘Chan–Paton factors’**) in the fundamental of some group G . This open string theory has spacetime gauge symmetry group G . (A massless spin one excitation with values in the adjoint of G appears.)

Unfortunately, it is also divergent at one loop where the string theory tries to emit massless closed string states. There are two such divergent one loop diagrams:



If we realise that the closed string sector appearing at one loop for this open string theory and the closed string we got from gauging Ω in the IIB theory are to be identified, we have a chance of cancelling these divergences against one another, leaving a consistent theory. The three divergent diagrams which occur are the Möbius strip, Klein bottle and cylinder:



It turns out that the divergences are given by a piece which is common to all of the amplitudes, times 32^2 for the Klein bottle, $\pm 2n_9 \times 32$ for the Möbius strip and n_9^2 for the cylinder, for Chan–Paton factors corresponding to a gauge group $SO(n_9)$ for the $-$ sign, and $USp(n_9)$ for the $+$ sign. This can be written as $(n_9 \pm 32)^2$, which cancels to zero for gauge group $SO(32)$.

We have thus recovered another chiral $\mathcal{N}=1$ spacetime supersymmetric string theory, with gauge group $SO(32)$, this time an open string theory.

We shall soon reinterpret this computation in the language of D–branes.

3.1. Massless fields

In types IIA and IIB string theory, there are the usual massless fields, G , Φ and B_2 , (graviton, dilaton and antisymmetric 2–form tensor) of a relativistic closed string, coming from the Neveu–Schwarz–Neveu–Schwarz (NS–NS) sector.

It is sometimes useful to think of the fundamental IIA or IIB string as carrying electric H_3 charge, where H_3 is the 3–form field strength of the potential B_2 . The two dimensional world-sheet of the string has a natural coupling to the 2–form potential B_2 . (Analogous to the familiar case of a charged particle coupling naturally to the 1–form Maxwell potential of electromagnetism via its world–line.)

There are also antisymmetric p –form fields A_p , coming from the Ramond–Ramond (R–R) sector. For type IIA there are A_1, A_3, A_5, A_7 and A_9 and for type IIB, A_0, A_2, A_4, A_6 and A_8 .

4. p –Branes

Imagine an extended object in the theory with p spatial dimensions. ($p=1$ for a string–like object). Imagine further that such an object arises if there is a $(p+1)$ –form in the theory to couple naturally to its $(p+1)$ –dimensional world volume.

Such objects are called **‘ p –branes’**[7].

So p –branes with p odd will arise in IIB string theory while for p even, they arise in Type IIA theory. They have (electric) charge μ_p with respect to the field strength F_{p+2} . Ten dimensional Hodge duality tells us that a p –brane also carries magnetic charge with respect to a form F_{8-2} and is therefore dual to a $(6-p)$ –brane.

A generalisation of Dirac’s argument[8] for electric and magnetic charges must apply here, setting the fundamental units of electric and magnetic charge.

5. D-Branes

It transpires[9] that there is a special class of p -branes which *carry the most basic unit of F_{p+2} charge allowed*: they are called ‘**D-branes**’. We may think of D-branes as the consequence of considering more general boundary conditions in string theory. The world-volume of the D-brane is the sub-manifold in spacetime upon which strings may have end-points. Specifically, the ‘D p -brane’ is defined by Neumann boundary conditions in $(p+1)$ directions (the world-volume) and Dirichlet in the remaining transverse directions.

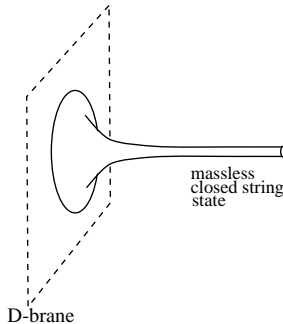
The pure open string theory we saw earlier is a very special example of a D-brane configuration: Spacetime itself contains D9-branes (with a ten dimensional world-volume). The (Chan-Paton) internal degree of freedom is the choice that a string endpoint has about which D9-brane it can end on. Gauge group $SO(32)$ corresponds to the presence of 32 D9-branes.

Think of the diagrams above as cylinders with either open string boundaries or crosscaps at either end. The boundary is a D9-brane.

6. The Divergences

The divergences come from the regions of moduli space where the cylinders’ lengths go to infinity. In this limit, the dominant contribution is from massless states in the closed string sector propagating.

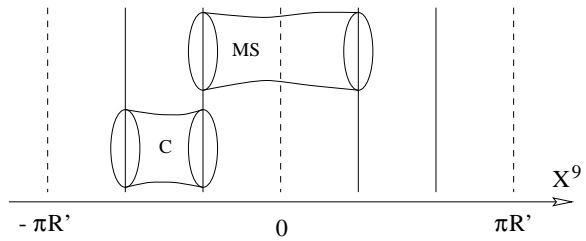
We may think[10,9] of the above cancellation as being between **tadpoles** coming from the emission of massless closed string states from a D9-brane (via the boundary term), or from the vacuum (via the crosscap).



This cancellation is associated with D9-branes. It is a requirement for consistency of the field equations of the 10-form R-R potential[10,11].

7. D-Branes and T -Duality

Another useful way to see the D-branes quite clearly is to imagine that the tenth dimension X^9 is a circle of radius R_9 . ‘ **T -duality**’ along X^9 gives an open string theory compactified on a circle of radius $R'=\alpha'/R_9$ which describes the same physics in new variables[12]. T -duality is a symmetry which can be described perturbatively as an action on the world sheet fields of the string theory[5]. One of its operations is to exchange Dirichlet with Neumann boundary conditions along the X_9 direction. This turns the 32 D9-branes into 32 D8-branes. Their world-volume is along the $\{X^0, \dots, X^8\}$ directions. Also, the action of Ω gets converted to a manifest spacetime reflection symmetry. The D8-branes have a transverse position coordinate on the X^9 circle (below, a line denotes the whole world-volume; a dotted line is an orientifold plane about which there is a reflection):



The one-loop diagrams seen earlier can be seen to arise clearly here as fundamental strings stretching between different branes. In the figure, the cylinder (C) is formed of strings stretching between distinct branes, while the Möbius strip (MS) comes from strings stretching between a brane and its mirror forming a one-loop diagram with a single boundary.

It also can be seen here is that $SO(32)$ is an enhanced gauge symmetry which occurs when the 32 branes and their images are all coincident. Moving them away from each other is equivalent to introducing Wilson lines in the dual picture to break some of the gauge symmetry.

A single isolated D-brane has gauge symmetry $U(1)$. Enhanced gauge symmetry arises when extra modes become massless, originating from strings stretched between coincident D-branes.

8. D-Branes as BPS States

An important class of states in supersymmetric theories are those which saturate a Bogomol'nyi-Prasad-Sommerfield[13] (**BPS**) bound. This is a lower bound on the mass of a state with respect to the central charges of the supersymmetry algebra. Well known examples of such states are furnished by the soliton sector of supersymmetric field theories.

There are a number of interesting and important properties of BPS states (see ref.[7] for more discussion and references):

- They break half the supersymmetry: Half of the supercharges annihilate them, the other half don't.
- The force between such BPS states is zero.
- The spectrum of masses and charges of BPS states in a given theory is exact: It may be computed at weak coupling, and then there is a non-renormalisation theorem which protects this spectrum from corrections, to all orders in perturbation theory.

D-Branes are BPS states[9]. They break half the supersymmetries, have zero force between each other, and their masses[5] ($\sim 1/g_{\text{str}}$) saturate the bound.

9. Application I: String/String Duality in Ten Dimensions

In the field theory examples of strong-weak coupling duality, one of the basic phenomena is the exchange of roles of the light, 'fundamental' charge carriers (electrons) with the heavy 'solitonic' dual degrees of freedom (monopoles), as the coupling goes large.

In a supersymmetric setting, these solitons will be BPS states whose properties we can study,

trusting that these properties will survive at strong coupling.

Many of the strong-weak coupling duality examples studied in the last year have urged us to identify the soliton in the theory which becomes light at strong coupling. It will carry the fundamental degrees of freedom in the dual theory.

In conjecturing strong-weak coupling duality relations between string theories, we can test the conjecture by studying the properties of the candidate soliton string which will become light at strong coupling. As each of the string theories has very distinct signature properties, it is easy to see if the conjecture has a good chance of being right, if one has the tools to look for these properties[14].

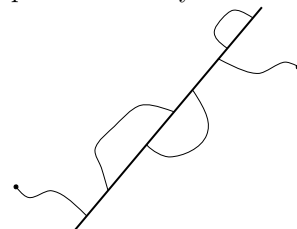
D-branes are the appropriate tools in many of these examples.

9.1. Type IIB Self Duality and $SO(32)$ Type I/Heterotic Duality

Two examples of a conjectured string-string duality in ten dimensions are the self duality of the type IIB string[15], and the duality relation between the $SO(32)$ string theories[16].

The obvious candidate soliton string which will become light at strong coupling in each theory is the D1-brane. As a soliton, it is very heavy at weak string coupling. However, its mass is inversely proportional to the string coupling, a basic property of D-branes[5], and so it will become light at strong coupling.

In both cases it has to have the right properties to become the dual string in the strong coupling limit: One can use basic D-brane calculus to study their world-sheet properties. The excitations of a D-brane can be studied by examining the fundamental strings which end on the brane, defining its position and dynamics:



Embedding such a macroscopic string in the background produces a decomposition of

the spacetime Lorentz group $SO(1,9)$ to $SO(8) \times SO(1,1)$. (The factors refer to directions transverse and parallel to the world-volume, respectively.)

The D1-brane in the type IIB theory has been shown to have right and left moving (with respect to $SO(1,1)$) fermions on its world-sheet which are of the same chirality $SO(8)$ spinors: the massless fermionic content of a IIB superstring[17].

Meanwhile, after projecting with Ω and adding D9-branes to get the type I theory, the D1-brane in that theory has only one $SO(8)$ spinor, coming from (say) the left, while the 1-9 and 9-1 fundamental strings produce a right-moving $SO(32)$ current algebra. This is the world sheet content of a heterotic string[18].

10. Application II: Extremal Black Hole Entropy

Another important application has been the demonstration that the Bekenstein–Hawking entropy/area law[19] of Black Hole Thermodynamics can be derived as a truly statistical result.

This was first done for five dimensional Reissner–Nordström black holes[20], and later shown for the four dimensional case[21,22].

The basic idea is simple[20]. The black holes of interest were embedded into (say) $K3 \times T^2$ compactified type IIB string theory as a BPS state. The abelian (Maxwell) fields they carry were embedded into the R-R sector of the compactified theory. The final embedding resembles a macroscopic string in six dimensions with a certain amount of R-R charge, together with a NS-NS charge representing momentum in one of the T^2 directions.

So far this is just a black hole dressed up with a stringy embedding. The area of its horizon can be computed, as normal, and thus the entropy may be computed using the Bekenstein–Hawking law. We can also compute all of the R-R and NS-NS charges it carries using ordinary flux integrals. The important point comes when we try to evaluate the entropy as a statistical quantity.

The internal structure of the black hole is a region we can only hope to probe with strong coupling string theory/quantum gravity. How-

ever, we can deform the theory to weak coupling (as the black hole is a BPS state of the string theory) and compute the entropy there. At weak coupling, the unique BPS saturated object in the string theory with the same quantum numbers as the black hole we saw at strong coupling, is a bound state of certain types of D-branes.

We can compute precisely how many D-branes are present in the composite due to the fact that we know the R-R charge of the composite, and the fact that D-branes carry the basic units of R-R charge[9].

The NS-NS charge corresponding to momentum is identified with the total momentum distributed among the fundamental strings connecting the constituent D-branes of the composite. Evaluating the number of ways to distribute this quantum number amongst the bound state’s constituents gives precisely the entropy.

11. Application III: Lower Dimensional $\mathcal{N}=1$ String Vacua

The basic orientifold example of the type I string can be extended to more complicated situations[4,23]. Developing the orientifold technology further, one can study a much larger class of string models than was previously possible with more traditional techniques. (The technology of constructing consistent string models in lower dimensions was largely limited to the closed string theories, as the unoriented and open string sectors were not as easily manipulated.)

The orientifold group is simply the group of symmetries of the string theory which we might like to consider gauging, in a way analogous to what was carried out for Ω in the ten dimensional example of section 3.

We consider discrete symmetries, which can be either purely of world-sheet (Ω) or of spacetime (like those used to form a conventional orbifold). Perturbative consistency of the models obtained by gauging such a group of symmetries is ensured by[24]:

- Finding a faithful representation of the symmetries of the orientifold group on the open string sectors (D-Branes).

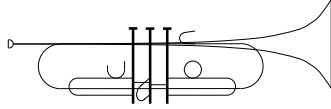
- Insisting on tadpole cancellation at one-loop.

In general, there will be tadpole cancellation associated with all the species of R-R $(p+1)$ -forms which can appear in the model, requiring the presence of a Dp -brane. Tadpole cancellation is another way of seeing that the R-R $(p+1)$ -form's field equation is satisfied.

In this way, one can study large families of $\mathcal{N}=1$ models in lower dimensions with interesting and important properties[4,23–27].

Constraints of time and space do not permit me to describe them further here.

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